Marine Technology and Engineering – Guedes Soares et al. (Eds)
© 2011 Taylor & Francis Group, London, ISBN 978-0-415-69808-5

Nonlinear ocean wave groups with high waves

F. Arena
Mediterranea University, NOEL, Reggio Calabria, Italy

ABSTRACT: Mechanics of three-dimensional wave groups, in a Gaussian sea, may be analyzed with the Quasi-Determinism (QD) theory, introduced by Boccotti in the eighties. This theory shows that, to the first order in a Stokes expansion, the structure of high waves is proportional to the autocovariance function. Formal derivation of QD theory up to the second-order was derived recently, for the mechanics of nonlinear ocean waves. This paper summarizes some results on linear and nonlinear wave groups, by considering effects of wave spectrum (variation of either frequency or directional spectra, for two- and three-dimensional waves respectively, including bimodal spectra) and of finite water depth. Finally, some results on mechanics of wave groups are discussed, by including diffracted fields: random wave groups interacting with a large vertical cylinder, with a horizontal submerged cylinder or a vertical wall, including nonlinear effects.

1 INTRODUCTION

To the first order in a Stokes expansion, the sea waves may be modelled by theory of wind-generated waves (Longuet-Higgins, 1963; Phillips, 1967) and free surface displacement and velocity potential represent a Gaussian process in time domain, at any fixed point.

The Quasi Determinism (QD) theory shows that in a Gaussian sea, if a very high wave occurs, the wave profile is a function of the autocovariance of the free surface displacement. The QD theory was developed in the eighties, in two formulations. The former (Boccotti, 1981, 1982, 1983, 2000) dealing with the crest height, proved that when at sea a very high crest H_C occurs at point (x_o, y_o) at time (t_o), the space-time profile of linear free surface displacement $(\bar{\eta}_1)$, with probability approaching 1, will be given by:

$$\bar{\eta}_1(x_o+X,\, y_o+Y,\, t_o+T)=\frac{\Psi(X,Y,T)}{\Psi(0,0,0)}H_C\,, \qquad (1)$$

where

$$\Psi(X,Y,T) \equiv <\eta(x_o,y_o,t)\;\eta\,(x_o+X,\,y_o+Y,\,t+T)>. \qquad (2)$$

The free surface displacement when a high crest occurs was also analyzed by Lindgren (1970, 1972) and by Tromans et al. (1991), who renamed the QD theory as NewWave model.

The latter formulation of the QD theory (Boccotti, 1989, 1993, 1997, 2000), dealing with the crest-to-trough wave height, proved that when a very large wave with height H occurs at point (x_o, y_o) at time (t_o), the linear free surface displacement $(\bar{\eta}_1)$ at any point $(x_o+X,\, y_o+Y,)$ at time (t_o+T), is given by:

$$\bar{\eta}_1(x_o+X,\, y_o+Y,\, t_o+T) =\frac{\Psi(X,Y,T)-\Psi(X,Y,T-T^*)}{\Psi(0,0,0)-\Psi(0,0,T^*)}\,\frac{H}{2}. \qquad (3)$$

The theory was verified with some small scale field experiments by Boccotti (1993, see also 2000 for a complete review). A field verification was also proposed by Phillips et al. (1993*a,b*).

The modeling of nonlinear random ocean waves was introduced by Longuet-Higgins (1963). Sharma & Dean (1979) proposed the second-order solution for ocean waves on an arbitrary water depth.

Second-order analysis of high wave groups was proposed in 2005 for long-crested waves. The solution up to the second order of the first formulation of the quasi determinism theory was proposed by Fedele & Arena (2005), with an analytical solution of the second-order crest height distribution; the mechanics of wave groups has been then analyzed in Arena and Fedele (2005). The more general solution for the second-order QD theory, in both the formulations, was then given by Arena (2005): in that paper long-crested waves in intermediate water depth were considered. The space-time evolution of nonlinear waves was investigated by Petrova, et al. (2010), with a validation proposed starting from data of the Draupner wave generated in a wave channel (Clauss et al., 2008).

Alternative approaches for nonlinear wave groups were proposed by Jensen (2005), based on the edgeworth form of Gram-Charlier series, and by Fedele and Tayfun (2009).

The generalization to three-dimensional waves in intermediate water was proposed recently (Arena et al, 2008), to analyze the group mechanics when a high wave occurs.

Effects of wave spectrum on the nonlinear properties of high random waves in sea states were investigated by Arena and Guedes Soares (2009*a,b*), by considering bimodal spectra.

Finally, the structure of wave groups in a non homogeneous wave field was given by Boccotti (1995, 1996, 2000). More recently, Romolo & Arena (2008, 2010, see also Romolo, 2006) investigated nonlinear standing waves, and Arena (2006) proposed the comparison between analytical predictions and experimental data of wave groups interacting with a submerged horizontal cylinder. Some results are summarized in this paper and the solution is proposed for random wave groups interacting with a large vertical cylinder.

2 QUASI-DETERMINISM THEORY FOR A GAUSSIAN SEA: THE UNDISTURBED WAVE FIELD

Here linear waves in a homogeneous field are considered. An example is represented by random waves in an undisturbed field. In this case, starting from linear theory of wind generated waves, it is possible to demonstrate that both free surface displacement and velocity potential represent Gaussian processes in time domain. Then, the QD theory may be applied in both formulations.

The QD theory, in its second formulation, was derived by starting from the probability density function of the surface displacement at point $(x_o+X,\, y_o+Y)$, at time t_o+T, given the condition

$$\eta(x_o, y_o, t_o)=\frac{1}{2}H, \quad \eta(x_o, y_o, t_o+T^*)=-\frac{1}{2}H \tag{4}$$

t_o being an arbitrary time instant, (x_o, y_o) an arbitrary point, H the crest-to-trough wave height and T^* the abscissa of the absolute minimum of the autocovariance function (2). Boccotti (1989, 1997, 2000) proved that, as $H/\sigma\to\infty$, condition (4) becomes both sufficient and necessary for the occurrence of a wave of given height H. As a consequence, as $H/\sigma\to\infty$, the linear free surface displacement tends asymptotically to the deterministic function (3).

In words we have that *"if a wave with a given height H occurs at a fixed point* (x_o, y_o) *and H is very large with respect to the mean wave height at this point, we may expect the water surface near* (x_o, y_o) *to be very close to thedeterministic form (3)"*.

The QD theory gives also the mechanics of the wave group: the linear velocity potential, when the large wave of height H occurs, is given by

$$\begin{aligned}&\bar{\phi}_1(x_o+X,\, y_o+Y,\, z,\, t_o+T)\\&\quad=\frac{\Phi(X,Y,z,T)-\Phi(X,Y,z,T-T^*)}{\Psi(0,0,0)-\Psi(0,0,T^*)}\,\frac{H}{2}\end{aligned} \tag{5}$$

where

$$\begin{aligned}&\Phi(X,Y,z,T)\\&\quad\equiv\langle\eta(x_o, y_o, t)\,\phi(x_o+X,\, y_o+Y,\, z,\, t+T)\rangle\end{aligned} \tag{6}$$

with X, Y, z, T the independent variables.

If we consider the first formulation of the QD theory, the velocity potential when a crest with height H_C occurs is given by:

$$\bar{\phi}_1(x_o+X,\, y_o+Y,\, z,\, t_o+T)=\frac{\Phi(X,Y,z,T)}{\Psi(0,0,0)}\,H_C\,, \tag{7}$$

with the linear free surface displacement given by Eq. (1). Equations (1) and (7) may be rewritten, as a function of the frequency or directional spectrum, as:

i) three-dimensional (short-crested) wave groups when a high crest occurs

$$\begin{aligned}&\bar{\eta}_1\left(x_0+X,\, y_0+Y,\, t_0+T\right)\\&\quad=\frac{H_C}{\sigma^2}\int_0^{\infty}\int_0^{2\pi}S(\omega,\theta)\\&\quad\cdot\cos\left(kX\sin\theta+kY\cos\theta-\omega T\right)\mathrm{d}\theta\,\mathrm{d}\omega\end{aligned} \tag{8}$$

$$\begin{aligned}&\bar{\phi}_1\left(x_0+X,\, y_0+Y,\, z,\, t_0+T\right)\\&\quad=\frac{H_C}{\sigma^2}g\cdot\int_0^{\infty}\int_0^{2\pi}S(\omega,\theta)\frac{\cosh\left[k(d+z)\right]}{\omega\cosh(kd)}\\&\quad\cdot\sin\left(kX\sin\theta+kY\cos\theta-\omega T\right)\mathrm{d}\theta\,\mathrm{d}\omega\end{aligned} \tag{9}$$

with $S(\omega,\theta)$ the directional spectrum;

ii) two-dimensional (long-crested) wave groups when a high crest occurs

$$\bar{\eta}_1(y_o+Y,\, t_o+T)=\frac{H_c}{\sigma^2}\int_0^{\infty}E(\omega)\cos(kY-\omega T)\,\mathrm{d}\omega \tag{10}$$

$$\bar{\phi}_1(y_o + Y, z, t_o + T) = g\frac{H_c}{\sigma^2}\int_0^\infty E(\omega)\omega^{-1} \cdot \{\cosh[k(d+z)]/\cosh(kd)\}\sin(kY - \omega T)\mathrm{d}\omega \tag{11}$$

with $E(\omega)$ the frequency spectrum.

Note that for three dimensional waves the variance of the free surface displacement is given by

$$\sigma^2 \equiv \int_0^\infty \int_0^{2\pi} S(\omega, \theta)\mathrm{d}\theta\mathrm{d}\omega \text{ 3D waves} \tag{12}$$

which, for two-dimensional waves, is reduced to

$$\sigma^2 \equiv \int_0^\infty E(\omega)\mathrm{d}\omega \text{ 2D waves} \tag{13}$$

Furthermore, the wave number k is related to the wave frequency ω by means of the linear dispersion rule:

$$k\tanh(kd) = \omega^2/g. \tag{14}$$

Finally, it should be noted that the QD theory is able to describe the space time evolution of a wave group when a high wave occurs, to the first order in a Stokes expansion. The velocity potential, given by either eq. (9) or eq. (11), for 3D or 2D waves respectively, gives also the wave kinematics. Then, if a slender cylinder is considered at sea, the Morison force given by high wave groups may be calculated by means of velocity potential of wave group. Examples were shown by Boccotti (2000), as well as by Arena & Romolo (2005), and by Arena & Nava (2008), where the comparison with the linearization approaches was proposed, for applications in offshore engineering.

3 QUASI-DETERMINISM THEORY FOR LINEAR WAVE DIFFRACTION

The quasi-determinism theory is valid whichever the boundary condition is. Then, it may be applied for a diffracted wave field too. Some examples were given by Boccotti (2000): waves interacting with a vertical wall (linear standing waves), or waves interacting with a semi-infinite wall. Other solutions were given by Arena (2006) for waves interacting with a horizontal submerged cylinder (see also Ogilvie, 1963 and Arena, 1999) and by Arena (1996) and Pavone & Arena (2004) for waves interacting with a single large vertical cylinder or an array of cylinders.

All these solutions were given for linear random waves: the wave field is non-homogeneous (the variance of free surface displacement changing from a point to each other), but both free surface displacement and velocity potential, at any fixed point, represent Gaussian processes in time domain. Here an example is shown for the random waves-large cylinder interaction. It is shown as the QD theory may be applied to the random wave force process too (as well as to the overturning moment).

3.1 *Random waves interacting with a large vertical cylinder*

The diffraction of random waves interacting with a large vertical cylinder may be solved starting from the solution of MacCamy and Fuchs (1954; see also Sumer and Fredose, 1987) for periodic waves, by using the general theory of the wind generated waves. Both the random free surface displacement and the random wave force represent, to the first order in a Stokes expansion, a Gaussian process. Then, the Quasi-Determinism theory may be applied: it is possible to calculate the free surface displacement when a high wave occurs on, or close to, the cylinder, as well as the force produced by this high wave.

3.1.1 *Incident wave field*

Free surface displacement, for the incident random wave field, may be written in a complex form as

$$\eta_i(r, \alpha, t) = \sum_{j=1}^{N} a_j \left\{ \sum_{m=0}^{\infty} \beta_m J_m(k_j r)\cos\left[m(\alpha - \theta_j)\right] \right\} \cdot \exp\left[-\mathrm{i}(\omega_j t + \varepsilon_j)\right]; \tag{15}$$

where the wave number k_i is related to the angular frequency ω_i by means of the linear dispersion rule (14), θ_i is the angle between the direction of propagation and the y-axis, $J_m(x)$ is the Bessel function of first kind, of integer order m and

$$\beta_o = 1, \quad \beta_m = 2i^m \quad \text{for} \quad m \geq 1$$

The reference frame is shown in Figure 1.

3.1.2 *Diffracted wave field*

Random free surface displacement, at any point (r,α), on or close to the vertical cylinder, may be written as (Arena, 1996):

$$\eta(r, \alpha, t) = \sum_{i=1}^{N} a_i \left[F(r, \alpha; \omega_i, \theta_i)\cos(\omega_i t + \varepsilon_i) + G(r, \alpha; \omega_i, \theta_i)\sin(\omega_i t + \varepsilon_i) \right], \tag{16}$$

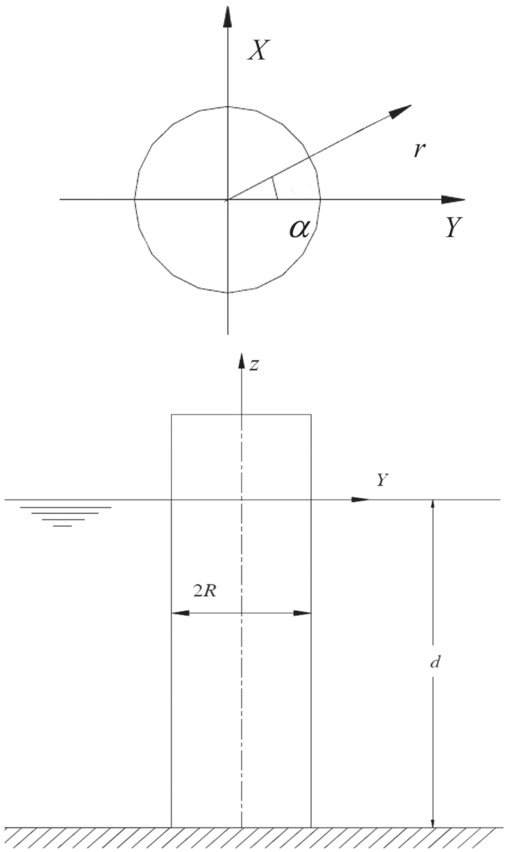

Figure 1. Reference frame for the analytical solution of random waves interacting with a large vertical cylinder. The plane *(X, Y)* is horizontal; the upward vertical axis z has origin at the mean water level.

where

$$F(r,\alpha;\omega_i,\theta_i)=\mathrm{Re}\left[B(r,\alpha;\omega_i,\theta_i)\right],$$
$$G(r,\alpha;\omega_i,\theta_i)=\mathrm{Im}\left[B(r,\alpha;\omega_i,\theta_i)\right], \tag{17}$$

$$B(r,\alpha;\omega_i,\theta_i)=\sum_{m=0}^{\infty}\beta_m\left[J_m(k_ir)-\frac{J_m'(k_iR)}{H_m^{(1)'}(k_iR)}H_m^{(1)}(k_ir)\right]\cos\left[m(\alpha-\theta_i)\right]. \tag{18}$$

Note that with $J_m(x)$ and $Y_m(x)$ are the Bessel function of first and second kind respectively. The Hankel function of the first kind is defined as $H_m^{(1)}(x)\equiv J_m(x)+\mathrm{i}\,Y_m(x)$.

The resulting wave field is then no homogeneous in space. The variance of the free surface displacement, at point (r, α), is

$$<\eta^2(r,\alpha,t)>=\sum_{i=1}^{N}\frac{1}{2}a_i^2\cdot\left[F^2(r,\alpha;\omega_i,\theta_i)+G^2(r,\alpha;\omega_i,\theta_i)\right] \tag{19}$$

or, as a function of the directional spectrum

$$<\eta^2(r,\alpha,t)>=\int_0^{\infty}\int_{-\pi}^{\pi}S(\omega,\theta)\cdot\left[F^2(r,\alpha;\omega,\theta)+G^2(r,\alpha;\omega,\theta)\right]\mathrm{d}\theta\,\mathrm{d}\omega. \tag{20}$$

The total random wave force on the cylinder, is obtained by integration of the wave pressure $\Delta p\left(R,\alpha;z,t\right)$ on the cylinder surface, as

$$f(t)=-\int_{-d}^{0}\int_{0}^{2\pi}\Delta p(R,\alpha;z,t)R\cos\alpha\,\mathrm{d}\alpha\,\mathrm{d}z \tag{21}$$

Solving the integral, the total random wave force is found, as a time function:

$$f(t)=4\rho g\sum_{i=1}^{N}a_i\frac{\tanh(k_id)}{k_i}\frac{D(k_iR)}{k_i}\cdot\cos\theta_i\cos(\omega_it+\varepsilon_i-\delta_i) \tag{22}$$

where

$$\delta_i=-\arctan\left[Y_1'(k_iR)/J_1'(k_iR)\right] \tag{23}$$

$$D(k_iR)=1/\sqrt{J_1'^2(k_iR)+Y_1'^2(k_iR)} \tag{24}$$

3.1.3 *Second formulation of quasi-determinism theory for the interaction between random waves and a large vertical cylinder*

Let us consider a large wave with height H occurring at point (r_o, α_o), with the crest of this high wave at time t_o. As H/σ tends to ∞, the probability tends to 1 that random free surface displacement at point $(r_o+\Delta r, \alpha_o+\Delta\alpha)$ at time t_o+T is given by:

$$\begin{aligned}\eta(r_o+\Delta r,\alpha_o+\Delta\alpha,t_o+T)\\=\frac{H}{2}\cdot\Big\{\big[&<\eta(r_o,\alpha_o,t)\eta(r_o+\Delta r,\alpha_o+\Delta\alpha,t+T)>\\&-<\eta(r_o,\alpha_o,t)\eta(r_o+\Delta r,\alpha_o+\Delta\alpha,t+T-T^*)>\big]\\&\Big/\big[<\eta^2(r_o,\alpha_o,t)>-<\eta(r_o,\alpha_o,t)\,\eta(r_o,\alpha_o,t+T^*)>\big]\Big\}\end{aligned} \tag{25}$$

or, as a function of the directional spectrum:

$$\eta(r_o+\Delta r, \alpha_o+\Delta\alpha, t_o+T)$$
$$= \frac{H}{2}\int_0^\infty\int_{-\pi}^{\pi} S(\omega,\theta)\cdot$$
$$\{[F_1F_2+G_1G_2][\cos(\omega T)-\cos(\omega(T-T^*))]$$
$$+[F_1G_2-F_2G_1][\sin(\omega T)-\sin(\omega(T-T^*))]\}\,\mathrm{d}\theta\mathrm{d}\omega$$
$$\Big/\int_0^\infty\int_{-\pi}^{\pi} S(\omega,\theta)[F_1^2+G_1^2][1-\cos(\omega T^*)]\,\mathrm{d}\theta\mathrm{d}\omega \tag{26}$$

with

$$\begin{cases} F_1 = F(r_o, \alpha_o; \omega_i, \theta_i); \\ G_1 = G(r_o, \alpha_o; \omega_i, \theta_i); \\ F_2 = F(r_o+\Delta r, \alpha_o+\Delta\alpha; \omega_i, \theta_i); \\ G_2 = G(r_o+\Delta r, \alpha_o+\Delta\alpha; \omega_i, \theta_i). \end{cases} \tag{27}$$

The wave force (22) represents a Gaussian process in time domain. Then, when the large wave occurs on the cylinder, or close to it, the QD theory enables to predict the wave force produced by this high wave. In detail, if at point (r_o, α_o), at time t_o a large wave with height H occurs, the total wave force on the cylinder at time t_o+T will be:

$$f(t_o+T) = \frac{H}{2}\{[<\eta(r_o,\alpha_o,t)f(t+T)>$$
$$-<\eta(r_o,\alpha_o,t)f(t+T-T^*)>]\}\Big/\{<\eta^2(r_o,\alpha_o,t)>$$
$$-<\eta(r_o,\alpha_o,t)\,\eta(r_o,\alpha_o,t+T^*)>\} \tag{28}$$

and, as a function of the directional spectrum:

$$f(t_o+T) = 2\rho g H\int_0^\infty\int_{-\pi}^{\pi} S(\omega,\theta)\frac{\tanh(kd)\,D(kR)}{k^2}$$
$$\cdot\cos\theta\{F_1[\cos(\omega T-\delta)-\cos(\omega(T-T^*)-\delta)]$$
$$-G_1[\sin(\omega T-\delta)-\sin(\omega(T-T^*)-\delta)]\}\mathrm{d}\theta\mathrm{d}\omega$$
$$\Big/\int_0^\infty\int_{-\pi}^{\pi} S(\omega,\theta)(F_1^2+G_1^2)[1-\cos(\omega T^*)]\,\mathrm{d}\theta\mathrm{d}\omega. \tag{29}$$

4 SECOND-ORDER QUASI-DETERMINISM THEORY FOR WAVE GROUPS IN AN UNDISTURBED FIELD

The solution for the second-order QD theory (Arena, 2005; Fedele & Arena, 2005; Arena et al., 2008) was given by following the perturbation method. The total second-order free surface displacement $\overline{\eta}$ and velocity potential $\overline{\phi}$ are given respectively by

$$\overline{\eta}(y_o+Y, t_o+T)$$
$$= \overline{\eta}_1(y_o+Y, t_o+T)+\overline{\eta}_2(y_o+Y, t_o+T)+o(H^2) \tag{30}$$

$$\overline{\phi}(y_o+Y, z, t_o+T)$$
$$= \overline{\phi}_1(y_o+Y, z, t_o+T)+\overline{\phi}_2(y_o+Y, z, t_o+T)+o(H^2) \tag{31}$$

where the second-order contribution is:

i) three-dimensional (short-crested) wave groups when a high crest occurs

$$\overline{\eta}_2 = \frac{H_C^2}{4\sigma^4}\int_0^\infty\int_0^\infty\int_0^{2\pi}\int_0^{2\pi} S(\omega_1,\theta_1)S(\omega_2,\theta_2)\{A_{1122}^{+}$$
$$\cdot\cos(\varphi_{11}+\varphi_{22})+A_{1122}^{-}\cos(\varphi_{11}-\varphi_{22})$$
$$+\delta_{12}2k_1\,/\sinh(2k_1 d)\}\mathrm{d}\theta_2\,\mathrm{d}\theta_1\,\mathrm{d}\omega_2\,\mathrm{d}\omega_1 \tag{32}$$

$$\overline{\phi}_2 = \frac{H_C^2}{4\sigma^4}\int_0^\infty\int_0^\infty\int_0^{2\pi}\int_0^{2\pi} S(\omega_1,\theta_1)S(\omega_2,\theta_2)\{B_{1122}^{+}$$
$$\cdot\cosh[k_{1122}^{+}(d+z)]/\cosh(k_{1122}^{+}d)\sin(\varphi_{11}+\varphi_{22})$$
$$+B_{1122}^{-}\frac{\cosh[k_{1122}^{-}(d+z)]}{\cosh(k_{1122}^{-}d)}\sin(\varphi_{11}-\varphi_{22})$$
$$-g\delta_{12}T2k_1\,/\sinh(2k_1 d)\}\mathrm{d}\theta_2\,\mathrm{d}\theta_1\,\mathrm{d}\omega_2\,\mathrm{d}\omega_1 \tag{33}$$

where

$$\varphi_{ij} \equiv k_i\sin\theta_j X + k_i\cos\theta_j Y - \omega_i T, \tag{34}$$

$$\delta_{12} = \begin{cases} 1 & \text{if } \omega_1 = \omega_2 \\ 0 & \text{otherwise.} \end{cases} \tag{35}$$

The expressions of interaction kernels $A_{ijlm}^{\pm}$ and $B_{ijlm}^{\pm}$ (Arena et al. 2008; see also Sharma and Dean, 1979) are:

$$A_{ijlm}^{\pm}(\omega_i,\theta_j,\omega_l,\theta_m) = \frac{B_{ijlm}^{\pm}(\omega_i\pm\omega_l)+(\omega_i^2+\omega_l^2)}{g}$$
$$-g\frac{\vec{k}_{ij}\cdot\vec{k}_{lm}}{\omega_i\omega_l}\pm\frac{\omega_i\omega_l}{g}, \tag{36}$$

$$B_{ijlm}^{\pm}(\omega_i,\theta_j,\omega_l,\theta_m) \equiv \frac{D_{ijlm}^{\pm}}{(\omega_i\pm\omega_l)}\frac{g^2}{\omega_i\omega_l} \tag{37}$$

$$D_{ijlm}^{\pm}(\omega_i,\theta_j,\omega_l,\theta_m)$$
$$\equiv \frac{\left[\sqrt{r_l}\,(k_i^2-r_i^2)\pm\sqrt{r_i}\,(k_l^2-r_l^2)\right](\sqrt{r_i}\pm\sqrt{r_l})}{(\sqrt{r_i}\pm\sqrt{r_l})^2-k_{ijlm}^{\pm}\tanh(k_{ijlm}^{\pm}d)}$$
$$+\frac{2(\sqrt{r_i}\pm\sqrt{r_l})^2[\vec{k}_{ij}\cdot\vec{k}_{lm}\mp r_ir_l]}{(\sqrt{r_i}\pm\sqrt{r_l})^2-k_{ijlm}^{\pm}\tanh(k_{ijlm}^{\pm}d)} \tag{38}$$

with

$$r_i\,(\omega_i) = \frac{\omega_i^2}{g} = k_i \tanh(k_i d) \tag{39}$$

$$\begin{aligned} &k_{ijlm}^{\pm}(\omega_i,\theta_j,\omega_l,\theta_m) \\ &\quad = \sqrt{k_i^2 + k_l^2 \pm 2k_i\,k_l\,\cos(\theta_j - \theta_m)}. \end{aligned} \tag{40}$$

Note that in the nonlinear velocity potential a term proportional to T has been considered, which derives from the Whitham (1974) discussion on the second-order problem of Stokes waves in finite depth (see also Dalzell, 1999; Boccotti 2000; Arena et al. 2008).

i) two-dimensional (long-crested) wave groups when a high crest occurs

$$\begin{aligned} \overline{\eta}_2(Y,t) = \frac{H_C^2}{4\sigma^4}\int_0^\infty \int_0^\infty \, & E(\omega_n)E(\omega_m) \\ & \cdot\left\{A_{nm}^{-}\cos\left(\varphi_n - \varphi_m\right)\right. \\ & \left. + A_{nm}^{+}\cos\left(\varphi_n + \varphi_m\right)\right\} \mathrm{d}\omega_m\, \mathrm{d}\omega_n \end{aligned} \tag{41}$$

$$\begin{aligned} \overline{\phi}_2(Y,\,z,\,T) = g^2\,\frac{H_C^2}{4\sigma^4}\int_0^\infty \int_0^\infty \, & E\,(\omega_n)E\,(\omega_m)\frac{1}{\omega_n} \\ \cdot\frac{1}{\omega_m}\left\{\frac{\cosh[k_{nm}^{-}(d+z)]}{\cosh(k_{nm}^{-}d)}\right. & \frac{B_{nm}^{-}\sin\left(\varphi_n - \varphi_m\right)}{\omega_n - \omega_m} \\ \left. + \frac{\cosh[k_{nm}^{+}(d+z)]}{\cosh(k_{nm}^{+}d)} \frac{B_{nm}^{+}\sin\left(\varphi_n + \varphi_m\right)}{\omega_n + \omega_m}\right\} & \mathrm{d}\omega_m\,\mathrm{d}\omega_n. \end{aligned} \tag{42}$$

where

$$\varphi(\omega_j,\,Y,\,T) \equiv k_j Y - \omega_j T \tag{43}$$

The interaction kernels $A_{ij}^{\pm}$ and $B_{ij}^{\pm}$ are (see Arena, 2005):

$$A_{nm}^{\pm} = \frac{B_{nm}^{\pm} - k_n k_m \pm \rho_n\rho_m}{\sqrt{\rho_n\rho_m}} + \rho_n + \rho_m \tag{44}$$

$$\begin{aligned} B_{nm}^{\pm} = \Big\{ & \left(\sqrt{\rho_n} \pm \sqrt{\rho_m}\right)\Big[\sqrt{\rho_m}\left(k_n^2 - \rho_n^2\right) \pm \sqrt{\rho_n}\,\big(k_m^2 \\ & -\rho_m^2\big)\Big] + 2\left(\sqrt{\rho_n} \pm \sqrt{\rho_m}\right)^2\left(k_n k_m \mp \rho_n\rho_m\right)\Big\} \\ & \Big/\left[\left(\sqrt{\rho_n} \pm \sqrt{\rho_m}\right)^2 - k_{nm}^{\pm}\tanh\left(k_{nm}^{\pm}d\right)\right] \end{aligned} \tag{45}$$

$$k_{nm}^{\pm} = |\,k_n \pm k_m \tag{46}$$

$$\rho_n = k_n \tanh(k_n d) = \omega_n^2 / g. \tag{47}$$

For deep water definitions of $A_{ij}^{\pm}$ and $B_{ij}^{\pm}$ are rewritten in the following simpler form (see Fedele and Arena, 2005):

$$A_{nm}^{-} = -\,|\,k_n - k_m\,|; \quad A_{nm}^{+} = k_n + k_m \tag{48}$$

$$B_{nm}^{+} = 0; B_{nm}^{-} = \frac{4k_n k_m\left(\sqrt{k_n} - \sqrt{k_m}\right)^2}{\left(\sqrt{k_n} - \sqrt{k_m}\right)^2 - |\,k_n - k_m\,|} \tag{49}$$

5 WAVE GROUPS WHEN A HIGH CREST-TO-TROUGH WAVE OCCURS: NONLINEAR SOLUTION FOR THE SECOND FORMULATION OF THE QD THEORY

The second formulation of the QD theory (Boccotti, 2000) shows what happens when a high crest-to-trough wave occurs at some fixed point and time. In this case the linear free surface displacement will tend to the deterministic profile (3) and the linear velocity potential to the eq. (5).

The nonlinear analysis was proposed by Arena (2005). If nonlinear free surface displacement is given by eq. (30), for long-crested waves, the linear and the second order contribution are written as:

$$\begin{aligned} \overline{\eta}_1(y_o + Y,\,t_o + T) = \frac{H}{2}\Big\{\int_0^\infty & E(\omega)\cos\left[\varphi(\omega,\,Y,\,T)\right] \\ -\cos\left[\varphi(\omega,\,Y,\,T) + \omega T^*\right]\mathrm{d}\omega\Big\} & \Big/\Big\{\int_0^\infty E(\omega) \\ \cdot\left[1 - \cos\left(\omega T^*\right)\right]\mathrm{d}\omega \Big\} & \end{aligned} \tag{50}$$

$$\begin{aligned} &\overline{\eta}_2(Y,T) \\ &\quad = \frac{H^2}{16}\left\{\int_0^\infty E(\omega)\left[1 - \cos\left(\omega T^*\right)\right]\mathrm{d}\omega\right\}^{-2} \\ &\quad\cdot\int_0^\infty \int_0^\infty E(\omega_n)E(\omega_m)\Big\{A_{nm}^{-}\Big[\big(1 - \cos(\omega_n T^*) \\ &\quad -\cos(\omega_m T^*) + \cos(\omega_n T^* - \omega_m T^*)\big)\cos\left(\varphi_n - \varphi_m\right) \\ &\quad + \left(\sin(\omega_n T^*) - \sin(\omega_m T^*) - \sin(\omega_n T^* - \omega_m T^*)\right) \\ &\quad \cdot\sin\left(\varphi_n - \varphi_m\right)\Big] + A_{nm}^{+}\Big[\big(1 - \cos(\omega_n T^*) - \cos(\omega_m T^*) \\ &\quad +\cos(\omega_n T^* + \omega_m T^*)\big)\cos\left(\varphi_n + \varphi_m\right) + \big(\sin(\omega_n T^*) \\ &\quad +\sin(\omega_m T^*) - \sin(\omega_n T^* + \omega_m T^*)\big) \\ &\quad \cdot\sin\left(\varphi_n + \varphi_m\right)\Big]\Big\}\;\mathrm{d}\omega_m\,\mathrm{d}\omega_n. \end{aligned} \tag{51}$$

6 SECOND-ORDER QUASI-DETERMINISM THEORY FOR WAVE GROUPS IN REFLECTION

The QD theory may be extended to include nonlinear effects for wave diffraction too. In this case, we should know the second-order solution for the random wave field in the presence of the body. An example is given by the recent solution for nonlinear reflection of high wave groups interacting with a vertical wall: it gives the second order QD solution for random standing waves. Then, if a large wave occurs at any point on the wall, or in front of it, we may determine at any point the free surface displacement and the velocity potential. As a consequence, the nonlinear pressure on the wall produced by the high wave group is known (see Romolo and Arena 2010 and Romolo, 2006 for long crested waves.

In detail, when an extremely high individual crest elevation H_C occurs at a fixed point (x_0, y_0) on or close to the wall, at a time instant t_0 in a random wind-generated sea state, which is assumed to be a stationary and Gaussian process of time, the linear 'Quasi-Determinism' theory predicts, with very high probability, the expected configuration of the wave field in time domain, before and after t_0, and in space domain, in a area surrounding (x_0, y_0). The free surface displacement and the velocity potential will be given by equations (1) and (7) respectively.

In this case, the covariance functions (2) and (6) will be referred to the linear random waves in reflection. If we define H_{C_R} the high crest occurring at (x_0, y_0), the deterministic linear free surface displacement is

$$\bar{\eta}_{1_R}(x_0+X, y_0+Y, t_0+T) = \frac{H_{C_R}}{\sigma^2}\int_{0_\omega}^{\infty}\int_{0_\theta}^{2\pi} S(\omega,\theta)\cos(kX\sin\theta-\omega T)\cos(k\,y_0\cos\theta)\cos[k(y_0+Y)\cos\theta]\,\mathrm{d}\theta\,\mathrm{d}\omega \tag{52}$$

and the velocity potential

$$\bar{\phi}_{1_R}(x_0+X, y_0+Y, z, t_0+T) = g\frac{H_{C_R}}{\sigma^2}\int_{0_\omega}^{\infty}\int_{0_\theta}^{2\pi} S(\omega,\theta)\frac{\cosh[k(d+z)]}{\omega\ \cosh(kd)}\sin(kX\sin\theta-\omega T)\cos(ky_0\cos\theta)\cos[k(y_0+Y)\cos\theta]\,\mathrm{d}\theta\,\mathrm{d}\omega \tag{53}$$

where $\sigma^2 \equiv \sigma^2(y_0)$ is 0.25 times the variance of the surface displacement of the random stationary and Gaussian wind-generated wave field in reflection as a whole, where the exceptionally high wave crest is realised; this is

$$\sigma^2(y_0) = \int_{0_\omega}^{\infty}\int_{0_\theta}^{2\pi} S(\omega,\theta)\cos^2(\,k\ y_0\cos\theta)\,\mathrm{d}\theta\,\mathrm{d}\omega \tag{54}$$

Note that, from the Bernoulli's equation, the linear wave pressure acting on the wall may be calculated as

$$\bar{p}_{w_{1_R}}(\underline{x}_0+\underline{X}, z, t_0+T) = \rho g\frac{H_{C_R}}{\sigma^2}\int_{0_\omega}^{\infty}\int_{0_\theta}^{2\pi} S(\omega,\theta)\frac{\cosh[k(d+z)]}{\cosh(kd)}\cos(kX\sin\theta-\omega T)\cos(ky_0\cos\theta)\cos[k(y_0+Y)\cos\theta]\,\mathrm{d}\theta\,\mathrm{d}\omega. \tag{55}$$

The second-order solution for the quasi determinism of random standing waves was derived recently for 2D and 3D waves (Romolo and Arena, 2010). It shows that second order contributions are given by

$$\begin{aligned}\bar{\eta}_{2_R}(\underline{x}_0+\underline{X}, t_0+T) = {} & H_{C_R}^2/8\ \sigma^4\int_{0_{\omega_1}}^{\infty}\int_{0_{\omega_2}}^{\infty}\int_{0_{\theta_1}}^{2\pi}\int_{0_{\theta_2}}^{2\pi} S(\omega_1,\theta_1)S(\omega_2,\theta_2)\cos(\varphi_1)\cos(\varphi_2)\\ & \cdot\{\{\mathrm{A}^-_{(\omega_1,\omega_2,\theta_1,\theta_2)_1}\cos(\alpha_1-\alpha_2)+\mathrm{A}^-_{(\omega_1,\omega_2,\theta_1,\theta_2)_2}\cos(\alpha_1+\alpha_2)\}\cos(\lambda_1\text{-}\lambda_2)+\{\mathrm{A}^+_{(\omega_1,\omega_2,\theta_1,\theta_2)_1}\\ & \cdot\cos(\alpha_1+\alpha_2)+\mathrm{A}^+_{(\omega_1,\omega_2,\theta_1,\theta_2)_2}\cdot\cos(\alpha_1-\alpha_2)\}\cos(\lambda_1+\lambda_2)\}\,\mathrm{d}\theta_2\,\mathrm{d}\theta_1\,\mathrm{d}\omega_2\,\mathrm{d}\omega_1-(\Xi/g)\end{aligned} \tag{56}$$

$$\begin{aligned}\bar{\phi}_{2_R}(\underline{x}_0+\underline{X}, z, t_0+T) = {} & g^2H_{C_R}^2/8\ \sigma^4\int_{0_{\omega_1}}^{\infty}\int_{0_{\omega_2}}^{\infty}\int_{0_{\theta_1}}^{2\pi}\int_{0_{\theta_2}}^{2\pi} S(\omega_1,\theta_1)S(\omega_2,\theta_2)\ \omega_1^{-1}\omega_2^{-1}\cos(\varphi_1)\cos(\varphi_2)\{\{\mathrm{C}^-_{(\omega_1,\omega_2,\theta_1,\theta_2)_1}\\ & \cdot\cos(\alpha_1-\alpha_2)+\mathrm{C}^-_{(\omega_1,\omega_2,\theta_1,\theta_2)_2}\cos(\alpha_1+\alpha_2)\}(\omega_1-\omega_2)^{-1}\sin(\lambda_1-\lambda_2)+\{\mathrm{C}^+_{(\omega_1,\omega_2,\theta_1,\theta_2)_1}\\ & \cdot\cos(\alpha_1+\alpha_2)+\mathrm{C}^+_{(\omega_1,\omega_2,\theta_1,\theta_2)_2}\cos(\alpha_1-\alpha_2)\}(\omega_1+\omega_2)^{-1}\sin(\lambda_1+\lambda_2)\}\,\mathrm{d}\theta_2\,\mathrm{d}\theta_1\,\mathrm{d}\omega_2\,\mathrm{d}\omega_1+\Xi T\end{aligned} \tag{57}$$

where the interaction kernels $A^{\mp}_{(\omega_1,\omega_2,\theta_1,\theta_2)\eta}$ and $C^{\mp}_{(\omega_1,\omega_2,\theta_1,\theta_2)\eta}$ are given by Romolo and Arena (2010). The linear $\overline{p}_{wR1}$ and the second-order $\overline{p}_{wR2}$ contributions of the wave pressure on a vertical wall may be calculated from Equations (53) and (57) respectively.

7 APPLICATIONS: LONG-CRESTED WAVES IN AN UNDISTURBED FIELD

Figure 2 shows the free surface displacement when a high wave occurs: the upper panel is obtained for the occurrence of a large crest-to-trough wave with height H, with $H/\sigma \to \infty$ (second formulation of QD theory); the lower panel is obtained for the occurrence of a large crest with height H_C with $H_C/\sigma \to \infty$ (first QD formulation). The comparison is shown between the linear wave groups η_1 and the total second-order free surface displacements $\eta_1 + \eta_2$. The waves are long crested, the spectrum is the mean JONSWAP (Hasselmann et al, 1973), and the water is deep. The nonlinearity produces crests higher and sharper and troughs lower and flatter. Furthermore, the upper panel shows that the crest-to-trough wave height is not modified by nonlinearity (Arena, 2005). In detail, if a large wave with height H occurs, from linear theory the profile η_1 is obtained, with both crest and trough amplitude of $0.5H$. To the second order, the total $\eta_1 + \eta_2$ profile, presents the crest amplitude of $0.57H$ and the trough amplitude of $0.43H$, so that the wave height is not modified.

The structure of high waves when a large crest occurs (lower panel) is symmetrical, with respect to the time $T = 0$, because it is proportional to the autocovariance function. For unimodal spectra, like JONSWAP, Pierson Moskowitz etc., the free surface displacement in time domain has a trend well described by lower panel of Figure 2. Strong modifications may occur for bimodal spectra, as shown by Arena & Guedes Soares (2009a, 2009b, 2010). Some examples are plotted in Figure 3, for bimodal spectra described following the Guedes Soares model (1984), with values of the sea state parameters given in Table 1. If compared with unimodal spectra, in general nonlinear effect may be either increased for mixed or wind-wave dominated seas or reduced for swell dominated seas. More details are given in the Arena & Guedes Soares (2009a,b), where analytical results, numerical simulations and ocean data were considered.

Figure 4 shows the space-time evolution of a 2D wave group: for some fixed time instant the wave group is shown in space domain. The group has the apex stage at time $T = 0$.

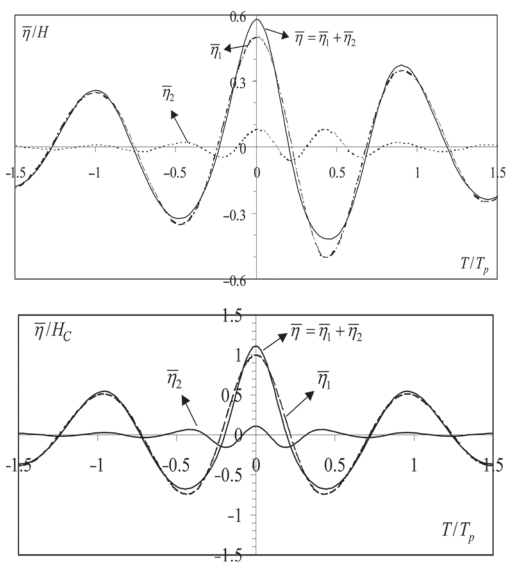

Figure 2. Let us assume that a wave (crest) with height H (H_C) occurs at ($Y = 0$), with $H/\sigma \to \infty$: the linear wave group $\overline{\eta}_1$, the second-order term $\overline{\eta}_2$ and the total second-order free surface displacements $\overline{\eta} = \overline{\eta}_1 + \overline{\eta}_2$, for the mean JONSWAP spectrum in deep water. Upper panel: high wave with height H; lower panel: high crest with height H_C.

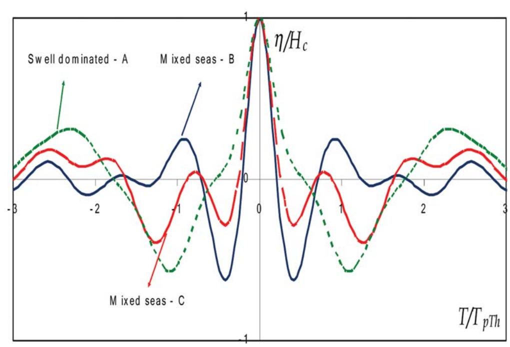

Figure 3. Let us assume that a crest with height H_C occurs: the free surface displacement for bimodal spectra shown in table 1 (Arena & Guedes Soares, 2009).

Table 1. Parameter defining the analyzed bimodal spectra of Figure 2 (Arena & Guedes Soares, 2009a).

Spectrum	H_s/L_z	H_R	T_R
A—Swell dominated sea	0.028	1.6	3.33
B—Mixed sea	0.067	0.8	1.43
C—Mixed sea	0.047	0.8	2.86

Note: $H_R \equiv H_{s_{sw}} / H_{s_{ww}}$ and $T_R \equiv T_{z_{sw}} / T_{z_{ww}}$.

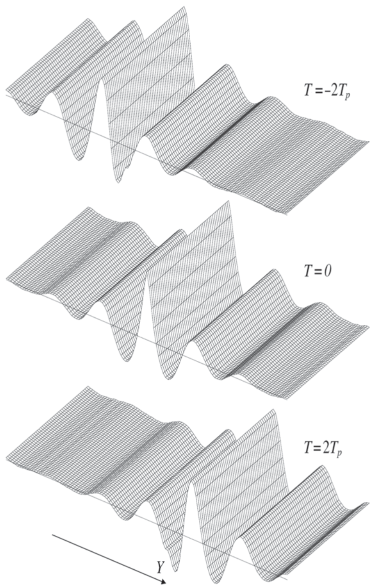

Figure 4. Two-dimensional (long-crested) waves: space time evolution of a wave group when a high wave occurs at a fixed point at $T = 0$. The water is deep and the dominant wave direction is given by Y-axis.

8 APPLICATIONS: SHORT-CRESTED WAVES IN AN UNDISTURBED FIELD

Nonlinear wave groups when a high crest occurs at sea, in a three-dimensional wave field, may be represented by means of Equations (8) and (32), for linear and second-order contribution respectively. Regarding the directional spectrum, the JONSWAP-Mitsuyasu is considered (Mitsuyasu et al, 1975).

A complete nonlinear analysis, for intermediate water depth, was proposed in Arena et al. (2008). Here a summary is given. Figure 5 shows the maximum nonlinear crest height when a linear crest of amplitude $H_{C\max}$ occurs in a three-dimensional wave field. The comparison with Figure 4 well represents the differences between 2D and 3D wave groups.

The maximum nonlinear amplitude increases as the water depth is reduced, whichever the spectrum is. By considering the bandwidth of the spectrum, it is found that in finite water depth the nonlinearity increases faster if a narrow-band model is considered (see Fig. 6). For finite bandwidth, by comparing short and long-crested waves, it is found that:

i) nonlinearity, in deep water, is slightly greater for long-crested waves;
ii) as the water depth is reduced, nonlinearity increases slower for long-crested than for short-crested waves

The conclusion is that for finite bandwidth of the spectrum, in intermediate and shallow water, the nonlinearity is stronger for three-dimensional waves. This conclusion was given in Arena et al. (2008) and is in full agreement with Forristall (2000), who analyzed data from numerical simulation for 2D and 3D waves, at different water depths. Comparison between the Forristall model for the crest height distributions in 2D and 3D waves, with the theoretical model given starting from the QD theory was given in Arena and Ascanelli (2010).

The wave front, along the X axis, is plotted in Figure 7, for different values of the n_p parameter

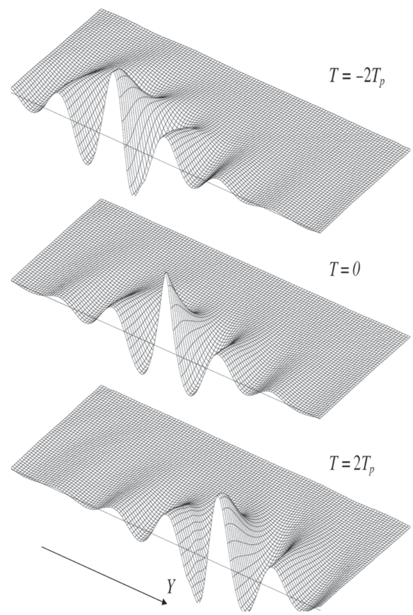

Figure 5. Three-dimensional (short-crested) waves: space time evolution of a wave group when a high wave occurs at a fixed point at $T = 0$. The water is deep and the dominant wave direction is given by Y-axis.

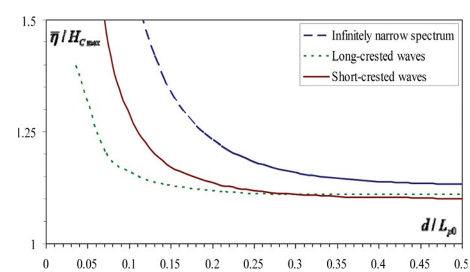

Figure 6. Effects of finite water depth for an infinitely narrow spectrum, for 2D and 3D waves (data from Arena et al., 2008).

of the Mitsuyasy's directional spreading function. For ocean waves a typical value of n_p is equal to 20. Then, the wave front has width equal to $2 \div 2.5$ times the wavelength in deep water L_{p0}. The waves tend to be long-crested as n_p increases, with the width of the wave front larger and larger.

Finally, it should be noted that QD theory gives the mechanics of the random wave groups, either to the first order or including nonlinear effects. The velocity potential under a very high crest in a three-dimensional wave field may be calculated by using Equations (9) and (32), for linear and second-order contribution respectively. The particle velocity and the acceleration may be then obtained and applied for calculation of the Morison force when a very high wave group occurs on a given slender cylinder.

Figure 8 plots the linear and the second order horizontal component of the velocity, v_y, when a very high crest occurs. Velocity v_y is plotted under the highest crest, as a function of water depth z, for deep water condition.

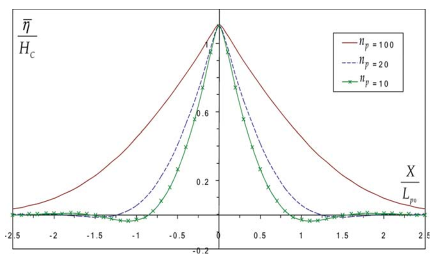

Figure 7. Second-order wave front (along X axis) when a high crest with elevation H_C occurs, for different value of the n_p parameter of the Mitsuyasu's directional spreading function (data from Arena et al, 2008).

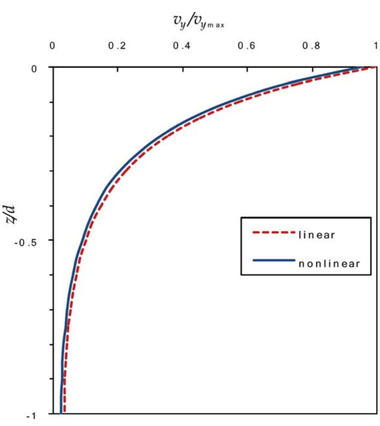

Figure 8. Linear and second-order horizontal velocity under a very high crest occurring in a 3D random wave field: the water is deep and the spectrum is the mean JONSWAP with the directional spreading function of Mitsuyasu.

9 APPLICATIONS: DIFFRACTED FIELDS

Quasi-determinism theory may be applied for diffracted fields too (Boccotti, 2000, 2008): it is valid whichever the boundary condition is.

An example of theoretical derivation of the QD theory for three dimensional waves interacting with a large vertical cylinder has been shown in section (3.1).

9.1 *Linear diffraction of random wave groups by a large vertical cylinder*

Figure 9 shows what happens when a high wave occurs at point A ≡ (R/L_p;π) on the cylinder: by applying Eq. (26) a wave group is obtained, which has been propagating along the dominant wave direction, coincident with the Y axis. The QD theory may be applied to determine the force process too. Figure 10 shows the force on the cylinder when the high wave shown in Figure 9 occurs on the cylinder. In detail, if a very large wave occurs at point A, fig. 10 shows the free surface displacement at points A, B ≡ (R/L_p; π/2) and C ≡ (R/L_p; 0). In the lower panel the wave force is shown in time domain.

Then, the QD theory enables us to analyze the mechanics of the three-dimensional waves interacting with a vertical cylinder. For example by analyzing how a large wave is generated at a fixed point on the cylinder. The wave force produced by this wave group, achieved from theoretical solution starting from the directional spectrum of the incident waves, gives a full description of the problem.

9.2 *Linear diffraction of random wave groups by a large horizontal cylinder*

The analytical solution for long-crested waves interacting with a horizontal submerged cylinder was given by Arena (2006), who started from the Ogilvie' solution (1963—see also Arena, 1999) for periodic waves. The wave pressure at any point on the cylinder, as well as the wave force, may be calculated from theoretical frequency spectrum. Figure 11 shows the wave pressures calculated on a cylinder and the wave forces, calculated from a mean JONSWAP spectrum, when a high wave occurs over the cylinder (data from Arena, 2006). This is a further property of the QD theory: because the force is a Gaussian process too, it may be applied to calculate the forces on the cylinder when a high wave occurs (for details see Boccotti, 2000; Arena, 2006).

9.3 *Nonlinear reflection of random waves*

The nonlinear reflection of random waves has been described in Section 6. Here some results

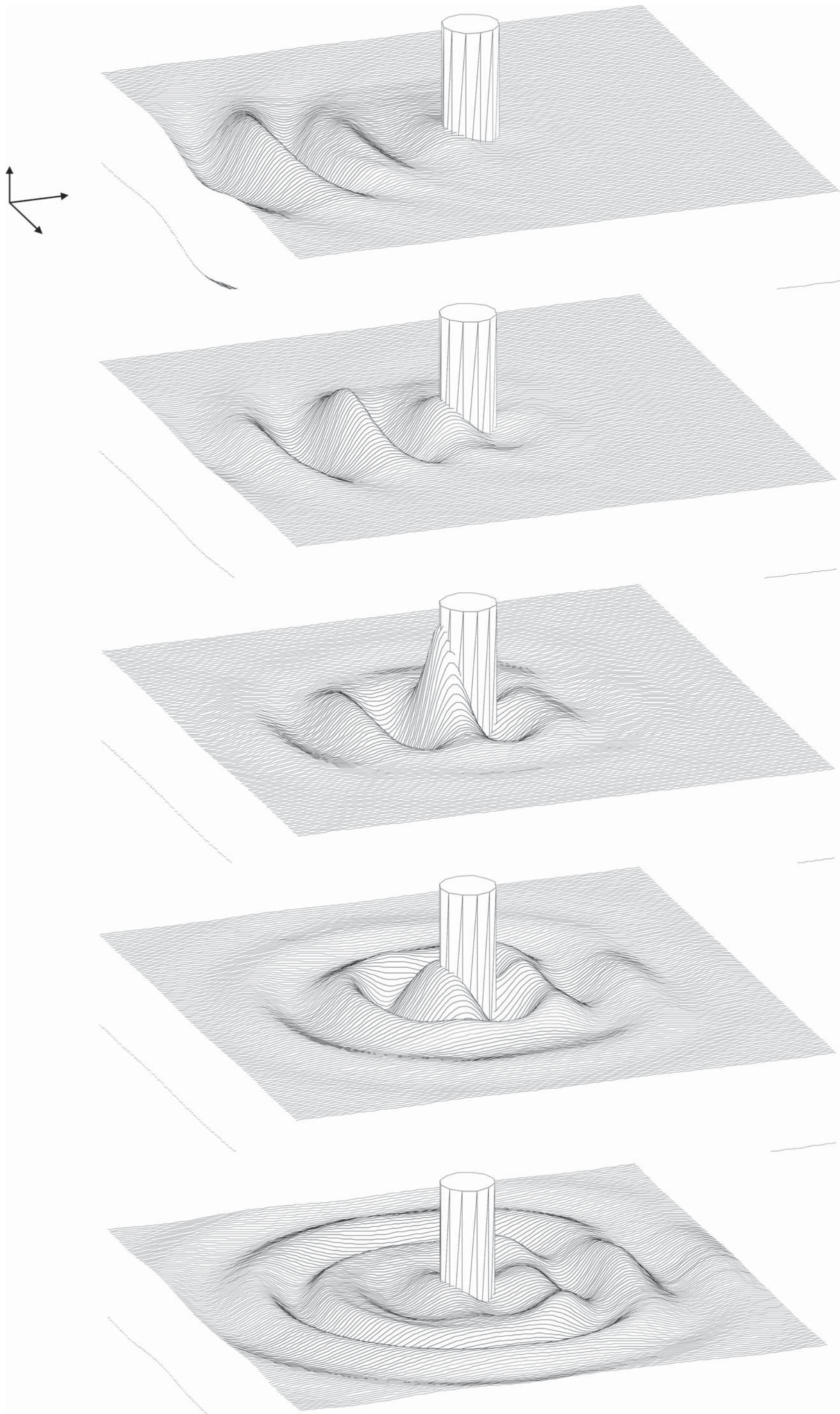

Figure 9. Let us consider a large crest-to-through wave height occurring at point A (R/L_{p0}, 180°) at time t_0: the three-dimensional wave group in which this wave occurs, at different times before and after t_0. The water is deep, the dominant wave direction is given by Y axis and $R = 0.3\ L_p$. Each plot shows a frame of length of 6 L_p along Y axis and of 4 L_p along X axis.

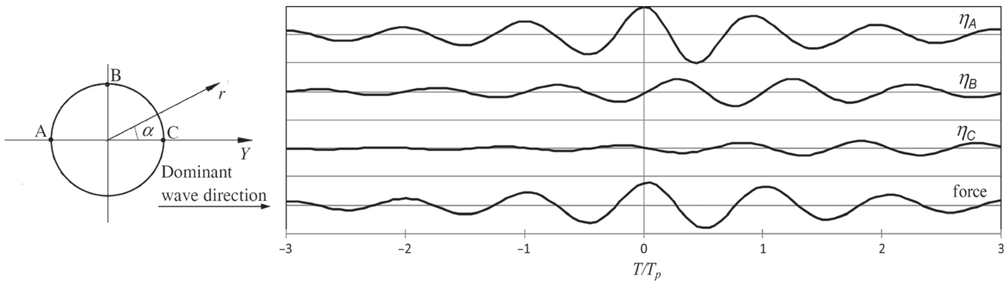

Figure 10. QD theory: let us suppose that a very large wave occurs at point A(R/L_p; π), with $R/L_p = 0.3$: the free surface displacement at points A, B ≡ (R/L_p; π/2), C ≡ (R/L_p; 0) and the wave force. The wave group in space-time domain is shown in Figure 9, with dominant wave direction given by Y axis..

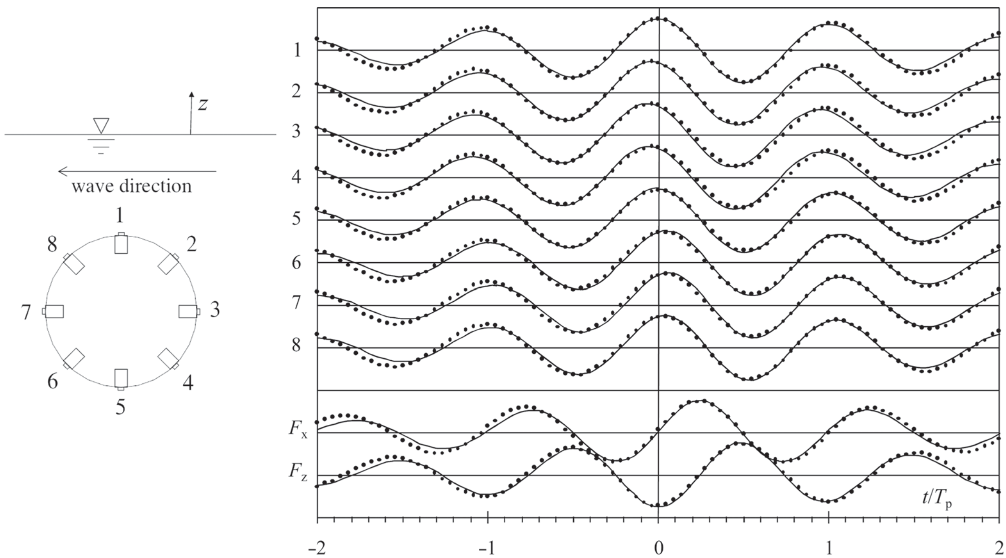

Figure 11. What happens when a very large height of the pressure fluctuation occurs top of a horizontal submerged cylinder. Comparison between experimental data (dotted lines) and analytical predictions (continuous lines), for radius of the cylinder $ka = 0.19$ and depth of the section centre $kh = 1.03$ (data recorded in NOEL, from Arena, 2006). F_x and F_z are the force components on the cylinder. On the left side the location of transducers (1–8) where the wave pressures were measured and calculated on a vertical section of the cylinder.

are shown. Figure 12, compares the free surface displacements $\overline{\eta}_1$ and $\overline{\eta}_1 + \overline{\eta}_2$, calculated either for the incident waves or for the wave reflection, at different values of the water depth (data from Romolo & Arena, 2008). As we can see, nonlinearity increases strongly for the standing waves; furthermore, it increases by reducing the water depth.

Data are achieved by considering a high crest that hits the wall, a mean JONSWAP spectrum and two values of the water depth d (including deep water condition). Note that the nonlinear effects in deep water (upper panels of Figure 12) are nearly doubled for wave reflection, with respect to waves in an undisturbed field.

Figure 13 shows the wave pressure on a vertical wall, when a high wave occurs. Wave pressure is calculated at the water depth $|z|$ equal to 0.5 L_{p0}. The nonlinear effects produce some characteristic modifications of the wave pressure, with a local minimum in the $\overline{\eta}_1 + \overline{\eta}_2$ profile occurring at the time of the wave crest of $\overline{\eta}_1$. The nonlinear profile may exhibits a crest with a double peak when a high crest occurs. These results are in full agreement with experimental evidence for reflection of ocean waves (see Boccotti, 2000; Arena and Fedele, 2002).

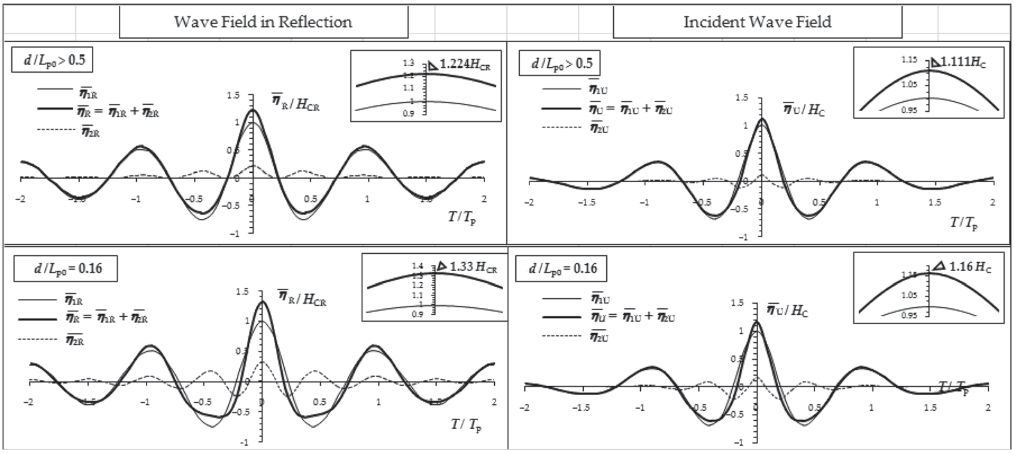

Figure 12. Standing waves compared to the incident waves (in an undisturbed field). Left side: free surface displacement on the vertical wall when a crest with height H_{CR} occurs on it, for two values of the water depth. Right side: free surface displacement of the incident waves. Comparison between linear $\overline{\eta}_1$, and the total second-order free surface displacements $\overline{\eta} = \overline{\eta}_1 + \overline{\eta}_2$. The mean JONSWAP spectrum is considered. Data from Romolo & Arena, 2008.

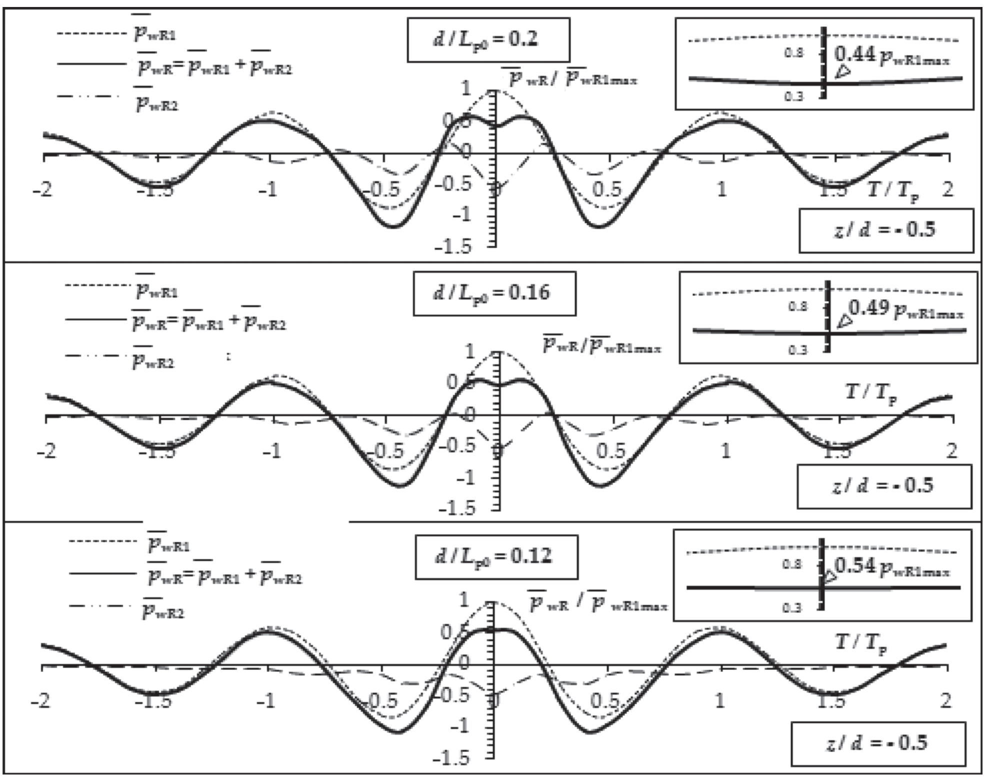

Figure 13. Wave pressure on a vertical wall, when a high wave occurs on it, calculated for $z/d = -0.5$. Comparison between linear $\overline{p}_{wR1}$, and the total second-order wave pressures $\overline{p}_{wR} = \overline{p}_{wR1} + \overline{p}_{wR2}$ for different values of the water depth d/L_{p0}. The mean JONSWAP spectrum is considered. Data from Romolo & Arena, 2006.

10 QUASI-DETERMINISM THEORY FROM FIELD DATA

Quasi determinism theory may be applied from field data. Some examples were given from small scale field experiments in the Natural Ocean Engineering Laboratory—NOEL, www.noel.unirc.it—of the Mediterranea University (Italy). In that lab, off the Reggio Calabria beach, a local wind from NNW often generates sea states consisting of pure wind waves with the typical size of a laboratory tank (significant wave height 0.20 m < H_s < 0.80 m, peak period 2.0 s < T_p < 3.6 s), with a small tide amplitude (typically within 0.15 m).

Usually, small-scale field experiments in NOEL (see, for example, Boccotti, 1995, 1996, 2000; Boccotti et al. 1993; Arena, 2006) the free surface displacements η and the fluctuating wave pressures η_{ph} (where $\eta_{ph} \equiv \Delta p / \rho g$), are recorded by means of ultrasonic probles and pressure transducers respectively. In most cases the sampling rate is equal to 10 Hz.

10.1 *Wave groups from wave data*

Let us consider data of free surface displacement recorded from two gauges in points A and B. If a very high crest H_c occurs in A, from Eq. (1) we have that linear free surface displacements in A is

$$\bar{\eta}_A(t_o + T) = H_C \Psi_A(T)/\Psi_A(0) \quad (58)$$

where

$$\Psi_A(T) = \langle \eta_A(t)\eta_A(t+T) \rangle \quad (59)$$

and that linear free surface displacements in B is

$$\bar{\eta}_B(t_o + T) = H_C \Psi_{AB}(T)/\Psi_A(0) \quad (60)$$

where

$$\Psi_{AB}(T) = \langle \eta_A(t)\eta_B(t+T) \rangle \quad (61)$$

Figure 14 shows a 300 s record of free surface displacement measured in May 2010 in the NOEL laboratory. It is considered as point A. A second gauge measured the free surface displacement at point B which was placed 1.5 m far from point A moving along X axis (which is assumed to be parallel to the shoreline).

Figure 15 shows the free surface displacement $\bar{\eta}_A$ at point A when a high crest occurs at that point: it is calculated by means of Eq. (58) by considering wave data of Figure 14.

Figure 16 shows also the free surface displacement $\bar{\eta}_B$ at point B when a high crest occurs at

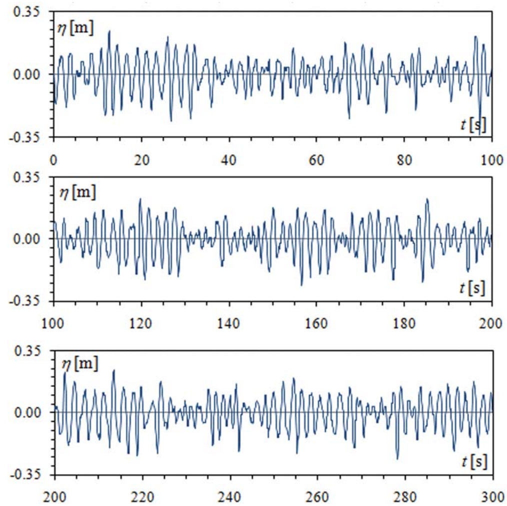

Figure 14. Record of free surface displacement in the NOEL laboratory: duration of 300 s with sampling frequency of 10 Hz.

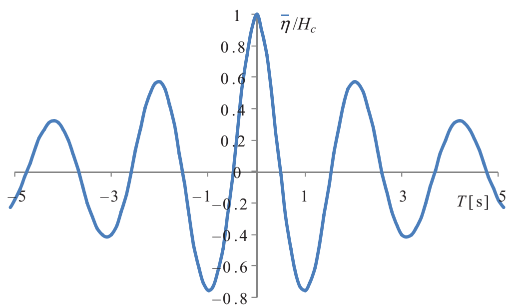

Figure 15. Free surface displacement when a high crest occurs, during the sea state plotted in Figure 14.

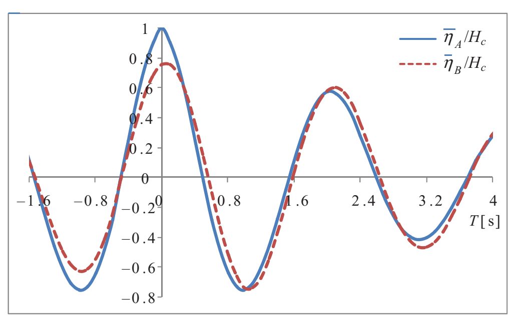

Figure 16. Let us consider a high crest occurring at point A, where free surface displacement of Figure 14 is measured: the free surface displacements at points A and B, calculated with Equations (58) and (60) respectively.

point A. Note that $\overline{\eta}_B$ is obtained from Eq. (60), with the cross covariance (61) calculated from free surface displacements recorded at points A and B simultaneously. From comparison between $\overline{\eta}_A$ and $\overline{\eta}_B$ it is possible to determine the dominant wave direction of the sea state: it will be a function of the phase shift and of the wave celerity, as shown by Boccotti (2000).

10.1 *Wave pressures and wave forces*

If a point C is considered, under the water surface, where a pressure transducer records the random process fluctuating wave pressure η_{ph}, given the high crest in A, we have

$$\overline{\eta}_{phC}(t_o + T) = H_C \Psi_{AC}(T) / \Psi_A(0), \tag{62}$$

where

$$\Psi_{AC}(T) = < \eta_A(t) \eta_{phC}(t+T) >. \tag{63}$$

With the same logic, QD theory may be applied to wave forces. An example is shown in Figure 11, where the wave pressures at transducers 1–8 and the wave forces F_y and F_z are plotted, when a high wave of the pressure at gauge 1 occurs.

11 CONCLUSIONS

In the paper the Boccotti's quasi-determinism theory for representing the ocean wave groups when a high wave occurs has been analyzed, either to the first order or including the second order contribution.

Firstly, the waves in an undisturbed field have been considered, analyzing both two-dimensional (long-crested waves), and three-dimensional wave groups as a function of the directional spectrum.

Nonlinear effects have been analyzed starting from the approaches proposed by Arena (2005) and Fedele & Arena (2005) (see also Arena & Guedes Soares, 2009a) for two-dimensional waves and by Arena et al (2008) for 3D waves.

Finally, the quasi-determinism theory has been applied to wave groups interacting with large structures. A large vertical cylinder has been considered, to analyze the diffraction effects, as well as a vertical wall, including second-order contributions. It has been shows as the theory may be applied to describe the mechanics of ocean waves and to determine the wave force on a structure produced by high waves.

LIST OF SYMBOLS

E	frequency spectrum
g	acceleration due to gravity
H	crest-to-trough wave height
H_C	crest amplitude
H_s	significant wave height
k	wave number
L	wavelength
S	directional spectrum
T_p	peak period
T	time
t_o	fixed time instant
x_o	fixed point of the x-axis
X	horizontal axis with origin at point x_o
y_o	fixed point of the y-axis
Y	horizontal axis with origin at point y_o
z	vertical coordinate axis, with the origin at the mean water level
α	wave amplitude
η	random free surface displacement
$\overline{\eta}$	surface displacement of deterministic wave groups
$\overline{\eta}_1$	linear component of $\overline{\eta}$
$\overline{\eta}_2$	second-order contribution of $\overline{\eta}$
η_{ph}	fluctuating wave pressures
ρ	water density
σ	r.m.s. surface displacement of a sea state
$\overline{\phi}$	velocity potential of deterministic wave groups
Φ	covariance of surface displacement and velocity potential
ψ^*	narrow bandedness parameter
Ψ	covariance of the surface displacement
θ	angle between wave direction and y-axis
ω	angular frequency
ω_p	peak frequency

REFERENCES

Arena, F. 1999. Note on a paper by Ogilvie: the interaction between waves and a submerged horizontal cylinder. *Journal of Fluid Mechanics*, 394: 355–356.

Arena, F. 2005. On non-linear very large sea wave groups. *Ocean Engng*, 32: 1311–1331.

Arena, F. 2006. Interaction between long-crested random waves and a submerged horizontal cylinder. *Physics of Fluids*, 18(7): paper 076602, 1–9.

Arena, F. & Fedele, F. 2005. Non-linear space-time evolution of wave groups with a high crest. *Journal of Offshore Mechanics and Ocean Engineering*, 127(1): 46–51.

Arena, F. & Romolo, A. 2005. Random Forces on a slender vertical cylinder given by high sea waves interacting with a current. *International Journal of Offshore and Polar Engineering*, 15(1): 21–27.

Arena, F. & Nava, V. 2008. On linearization of Morison force given by high three-dimensional sea wave groups, *Probabilistic Engineering Mechanics.* 23(2–3): 104–113.

Arena, F. & Guedes Soares, C. 2009*a*. 'Nonlinear high wave groups in bimodal sea states,' *ASCE Journal of Waterway, Port, Coastal, and Ocean Engineering*, 135(3): 69–79.

Arena, F. & Guedes Soares, C. 2009*b*. Nonlinear crest, trough and wave height distributions in sea states with double-peaked spectra, *ASME Journal of Offshore Mechanics and Ocean Engineering*. 131(4): 1–8. paper 041105.

Arena, F. & Ascanelli, A. 2010. Nonlinear crest height distribution in three-dimensional waves, *ASME Journal of Offshore Mechanics and Ocean Engineering*. 132(2): 1–6, paper 021604–5.

Arena, F. & Guedes Soares, C. 2008. On the sequence of high waves in nonlinear groups, *Proc. of the 27th International Conference on Offshore Mechanics and Arctic Engineering—OMAE 2008—of ASME, Estoril, Portugal, 15–20 June*

Arena, F., Ascanelli, A., Nava, V., Pavone, D. & Romolo, A. 2008. Non-linear three-dimensional wave groups in finite water depth, *Coastal Engineering*, 55(12): 1052–1061.

Boccotti, P. 1981. On the Highest Waves in a Stationary Gaussian Process, *Atti Acc Ligure di Scienze e Lettere*, 38: 271–302.

Boccotti, P. 1982. On Ocean Waves with High Crests, *Meccanica*, 17: 16–19.

Boccotti, P. 1983. Some New Results on Statistical Properties of Wind Waves, *Applied Ocean Research*, 5: 134–140.

Boccotti, P. 1989. On Mechanics of Irregular Gravity Waves, *Atti Acc Naz Lincei, Memorie*, 19: 11–170.

Boccotti, P., 1995. A Field Experiment on the Small-Scale Model of a Gravity Offshore Platform, *Ocean Eng.,* 22: 615–627.

Boccotti, P., 1996. Inertial Wave Loads on Horizontal Cylinders: A Field Experiment, *Ocean Eng.,* 23: 629–648.

Boccotti, P. 1997. A General Theory of Three-dimensional Wave Groups, *Ocean Engineering*, 24: 265–300.

Boccotti, P. 2000. *Wave mechanics for ocean engineering*, Elsevier Science, Oxford, 1–496.

Boccotti, P. 2008. Quasideterminism theory of sea waves, *ASME Journal of Offshore Mechanics and Arctic Engineering*, 130(4): 1–9, paper 041102.

Boccotti, P., Barbaro, G. & Mannino, L. 1993. A field experiment on the mechanics of irregular gravity waves, *J. Fluid Mech.* 252: 173 .

Clauss, G., Klein, M. & Testa, D. 2008. Spatial evolution of an extreme sea state with an embeded rogue wave. In *Proceedings of the 27th International Conference on Offshore Mechanics and Arctic Engineering OMAE 2008*, Estoril, Portugal.

Dalzell, J.F. 1999. A note on finite depth second-order wave-wave interactions, *Applied Ocean Research*, 21: 105–111.

Fedele, F. & Arena, F. 2005. Weakly nonlinear statistics of high random waves. *Phys. Fluids,* 17: 1–10, paper 026601.

Fedele, F. & Tayfun, M.A. 2009. On nonlinear wave groups and crest statistics. *J. Fluid Mech. 6*20: 221–239.

Forristall, G.Z. 2000. Wave Crest Distributions: Observations and Second-Order Theory. *Journal of Physical Oceanography*, 30: 1931–1943.

Guedes Soares, C. 1984. Representation of Double-Peaked Sea Wave Spectra, *Ocean Engng*, 11: 185–207.

Hasselmann, K., Barnett, T.P. & Bouws, E., et al. 1973. Measurements of wind wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP). *Dtsch. Hydrogr. Zeit.* A8: 1–95.

Jensen, J.J. 2005. Conditional second-order short-crested water waves applied to extreme wave episodes. *J. Fluid Mech.*, 545: 29–40.

Lindgren, G. 1970. Some properties of a normal process near a local maximum, *Ann. Math. Stat.* 41: 1870.

Lindgren, G. 1972. Local maxima of Gaussian fields, *Arkiv Feur Matematik* 10: 195.

Longuet-Higgins M.S. 1963. The effects of non-linearities on statistical distributions in the theory of sea waves. *Journal of Fluid Mechanics*, 17: 459–480.

MacCamy, R.C. & Fuchs, R.A., 1954. Wave Forces on Piles: A Diffraction Theory. U.S. Army Corps of Engineers, Beach Erosion Board, Tech. Memo N°69.

Mitsuyasu H., Tasai F. & Suara T. 1975. Observation of Directional Spectrum of Ocean Waves Using a Clover-Leaf Buoy. *Journal of Physical Oceanography*, 5: 750–760.

Ogilvie T.F. 1963. First- and second-order forces on a cylinder submerged under a free surface, *J. Fluid Mech.* 16: 451–472.

Pavone D. & Arena, F. 2006. Forces Of Random Wave Groups On Vertical Cylinders. In *Proc. of the 25th International Conference on Offshore Mechanics and Arctic Engineering—OMAE 2006—of ASME*, Hamburg, Germany, paper 92484, 1–5.

Petrova, P., Arena, F. & Guedes Soares, C. 2010. Space-time evolution of random wave groups with high waves based on the quasi-determinism theory, under peer review.

Phillips, O.M. 1967. The theory of wind-generated waves. *Advances in Hydroscience*, 4: 119–149.

Phillips O.M., Gu D. and Donelan M. 1993*a*. On the expected structure of extreme waves in a Gaussian sea, I. Theory and SWADE buoy measurements. *J. Phys. Oceanogr.*, 23: 992–1000.

Phillips O.M., Gu D. & Walsh E.J. 1993*b*. On the expected structure of extreme waves in a Gaussian sea, II. SWADE scanning radar altimeter measurements. *J. Phys. Oceanogr.*, 23: 2297–2309.

Romolo, A. 2006. Mechanics of nonlinear random sea waves interacting with a reflective upright breakwater: Formal derivation and validation, Ph.D. thesis, Mediterranea University of Reggio Calabria, 1–291.

Romolo, A. & Arena, F. 2008. Mechanics of nonlinear random wave groups interacting with a vertical wall, *Physics of Fluids.* 20(3): 1–16, paper 036604.

Romolo, A. & Arena F. 2010. A theory on reflection of nonlinear short-crested wave groups. In: *Atti XXXII Convegno Nazionale di Idraulica e Costruzioni Idrauliche*, Palermo, 14–17 settembre, paper B0482, 1–10.

Sharma J.N. & Dean R.G. 1979. Development and Evaluation of a Procedure for Simulating a Random Directional Second-order Sea Surface and Associated Wave Forces. *Ocean Engineering Rep. 20, University of Delaware*, 1–112.

Sumer B.M. & Fredsoe J. 1997. *Hydrodynamics around cylindrical structures.* Adv Series on Ocean Eng, World Scientific.

Tromans P.S., Anaturk A.R. & Hagemeijer P. 1991. A new model for the kinematics of large ocean waves—application as a design wave—. *Shell International Research*, publ. 1042.

Whitham G.B. 1974. Linear and Nonlinear Waves. New York: Wiley.