$$= \log\frac{|t|}{|t+1|} + c \quad = \log\frac{e^x}{e^x+1} + c$$

Pertanto

$$\int_1^{+\infty}\frac{1}{e^x+1}\,dx = \lim_{p\to+\infty}\int_1^{p}\frac{1}{e^x+1}\,dx = \lim_{p\to+\infty}\left[\log\frac{e^x}{e^x+1}\right]_1^p =$$

$$= \lim_{p\to+\infty}\left[\overset{\to 1}{\log\frac{e^p}{e^p+1}} - \log\frac{e}{e+1}\right] = -\log\frac{e}{e+1} = \log\left(1+\frac{1}{e}\right)$$

6) Si tratta di una serie a t. positivi – Per studiarne il carattere applichiamo il criterio della radice e calcoliamo

$$\lim_{n\to\infty}\sqrt[n]{\frac{5^n}{n^4\cdot 8^{2n}}} = \frac{5}{64}\cdot\lim_{n\to\infty}\frac{1}{(\underbrace{\sqrt[n]{n}}_{\to 1})^4} = \frac{5}{64} < 1$$ dunque la serie converge

per il cor. del t. sulle succ.: $\{n\}_{n\in N}$

Poiché $\lim_{n\to\infty}\frac{n+1}{n} = 1 \Rightarrow \lim_{n\to\infty}\sqrt[n]{n} = 1$

7) $|z| = \sqrt{4+4} = \sqrt{8}$

$$\begin{cases}\cos\vartheta = \frac{2}{\sqrt8} = \frac{\cancel{2}}{\cancel{2}\sqrt2} = \frac{\sqrt2}{2} \\ \mathrm{sen}\,\vartheta = -\frac{2}{\sqrt8} = -\frac{\sqrt2}{2}\end{cases} \Rightarrow \vartheta = -\frac{\pi}{4} \quad \Rightarrow z = \sqrt8\left(\cos\left(-\frac{\pi}{4}\right) + i\,\mathrm{sen}\left(-\frac{\pi}{4}\right)\right)$$

Dunque se $W^4 = z$, le radici quarte di z saranno:

$$W_0 = \sqrt[8]{8}\left(\cos\left(-\frac{\pi}{16}\right) + i\,\mathrm{sen}\left(-\frac{\pi}{16}\right)\right)$$

$$W_1 = \sqrt[8]{8}\left(\cos\frac{-\frac{\pi}{4}+2\pi}{4} + i\,\mathrm{sen}\left(\frac{7}{16}\pi\right)\right) = \sqrt[8]{8}\left(\cos\frac{7}{16}\pi + i\,\mathrm{sen}\frac{7}{16}\pi\right)$$

$$W_2 = \sqrt[8]{8}\left(\cos\frac{-\frac{\pi}{4}+4\pi}{4} + i\,\mathrm{sen}\frac{15}{16}\pi\right) = \sqrt[8]{8}\left(\cos\frac{15}{16}\pi + i\,\mathrm{sen}\frac{15}{16}\pi\right)$$

$$W_3 = \sqrt[8]{8}\left(\cos\frac{-\frac{\pi}{4}+6\pi}{4} + i\,\mathrm{sen}\frac{23}{16}\pi\right) = \sqrt[8]{8}\left(\cos\frac{23}{16}\pi + i\,\mathrm{sen}\frac{23}{16}\pi\right)$$